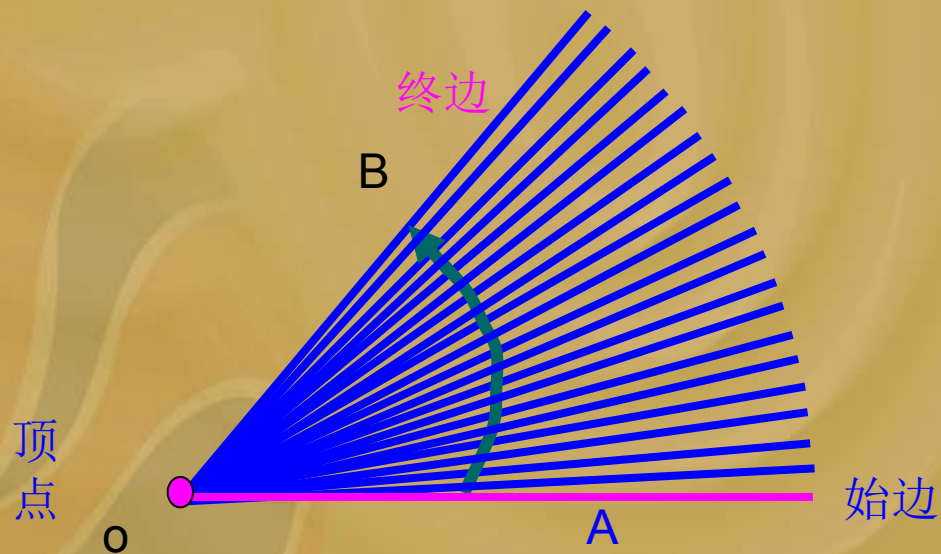
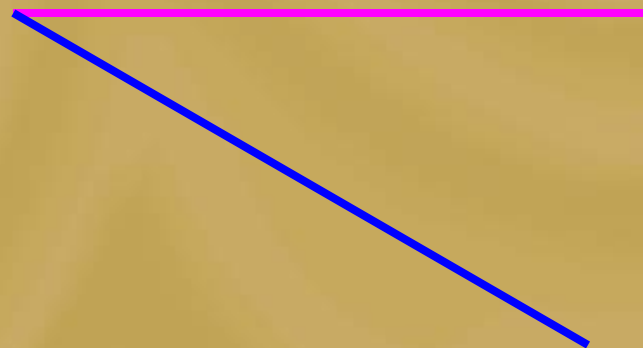


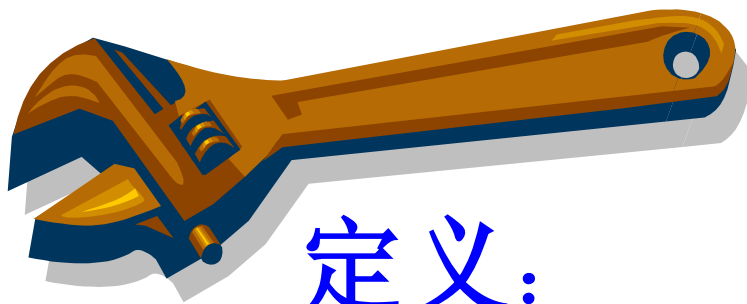
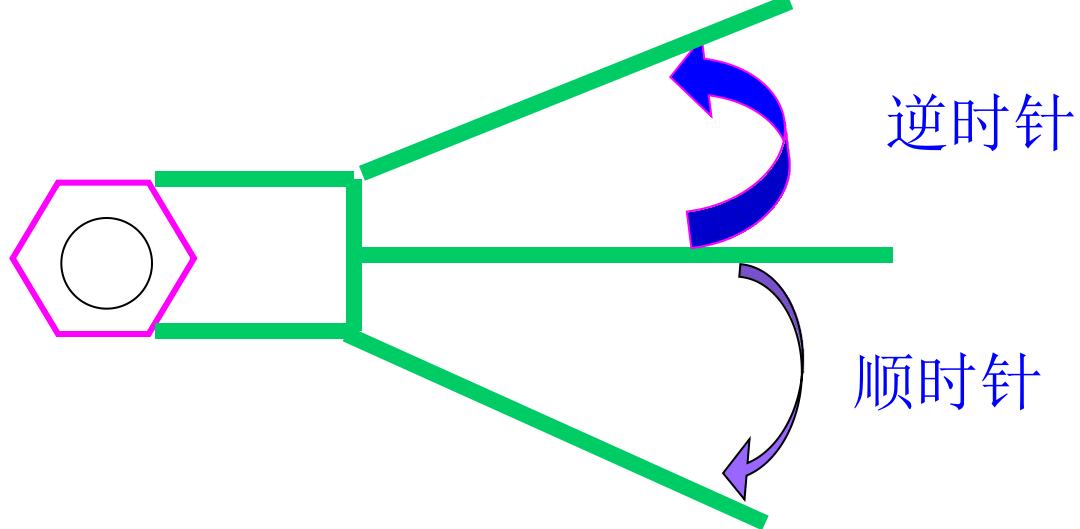
角的概念的推广 (2)





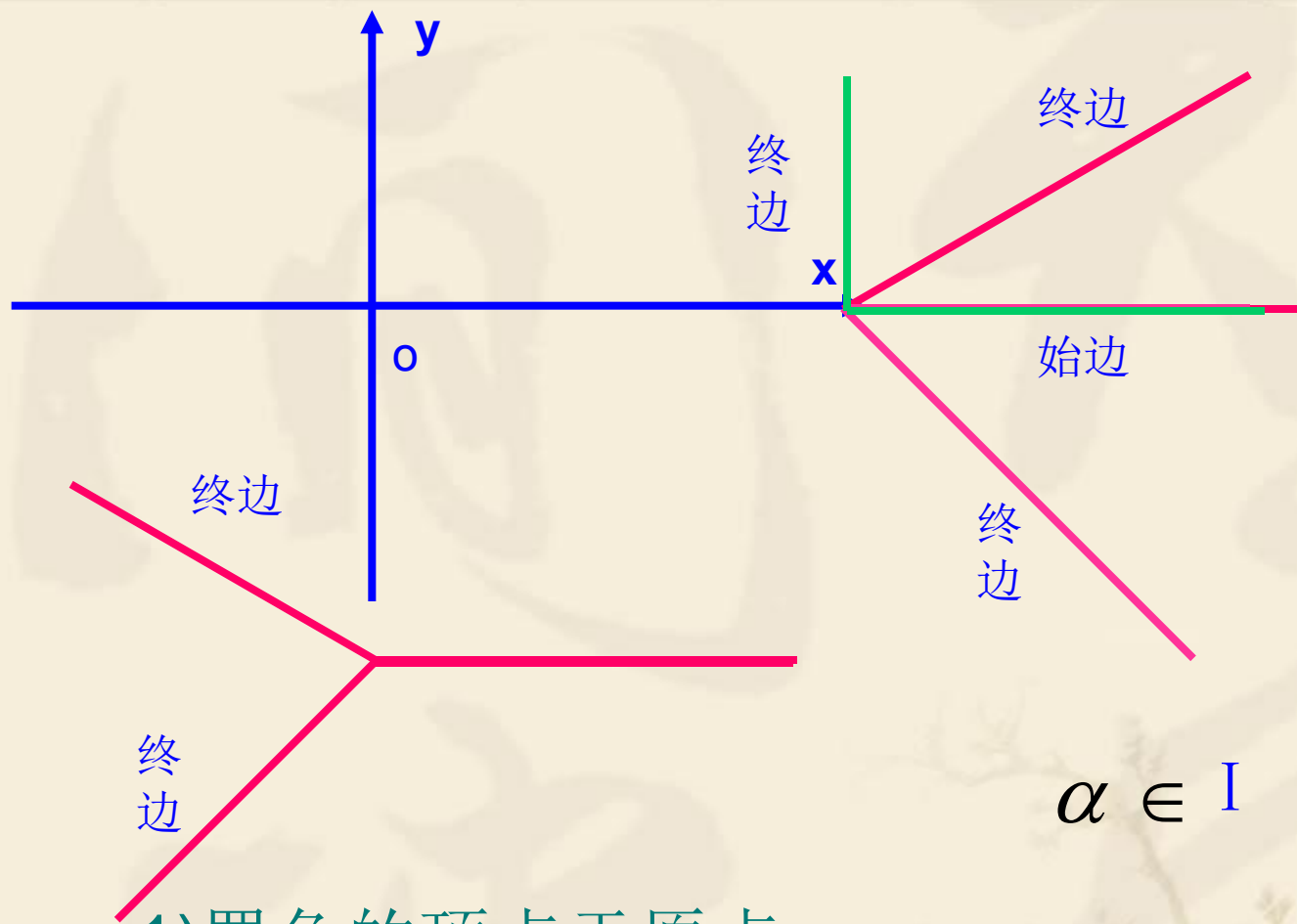
角：一条射线绕着它的端点在平面内旋转形成的图形





定义：

任意角 { 正角：按逆时针方向旋转形成的角
负角：按顺时针方向旋转形成的角
零角：射线不做旋转时形成的角



$$\alpha \in \text{I}$$

$$\alpha \in \text{II}$$

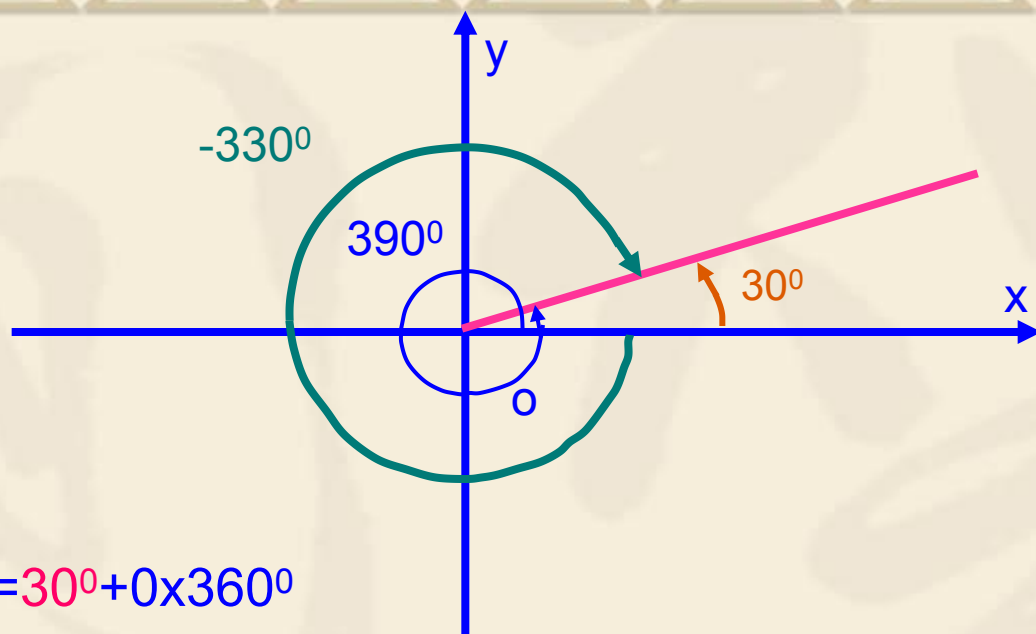
$$\alpha \in \text{III}$$

$$\alpha \in \text{IV}$$

1)置角的顶点于原点

2)始边重合于X轴的正半轴

终边落在第几象限就是第几象限角



$$30^{\circ} = 30^{\circ} + 0 \times 360^{\circ}$$

$$390^{\circ} = 30^{\circ} + 360^{\circ} = 30^{\circ} + 1 \times 360^{\circ}$$

$$-330^{\circ} = 30^{\circ} - 360^{\circ} = 30^{\circ} - 1 \times 360^{\circ}$$

$$30^{\circ} + 2 \times 360^{\circ}, \quad 30^{\circ} - 2 \times 360^{\circ}$$

$$30^{\circ} + 3 \times 360^{\circ}, \quad 30^{\circ} - 3 \times 360^{\circ}$$

...

...

与 30° 终边相同的角的一般形式为 $30^{\circ} + K \times 360^{\circ}$, $K \in \mathbb{Z}$

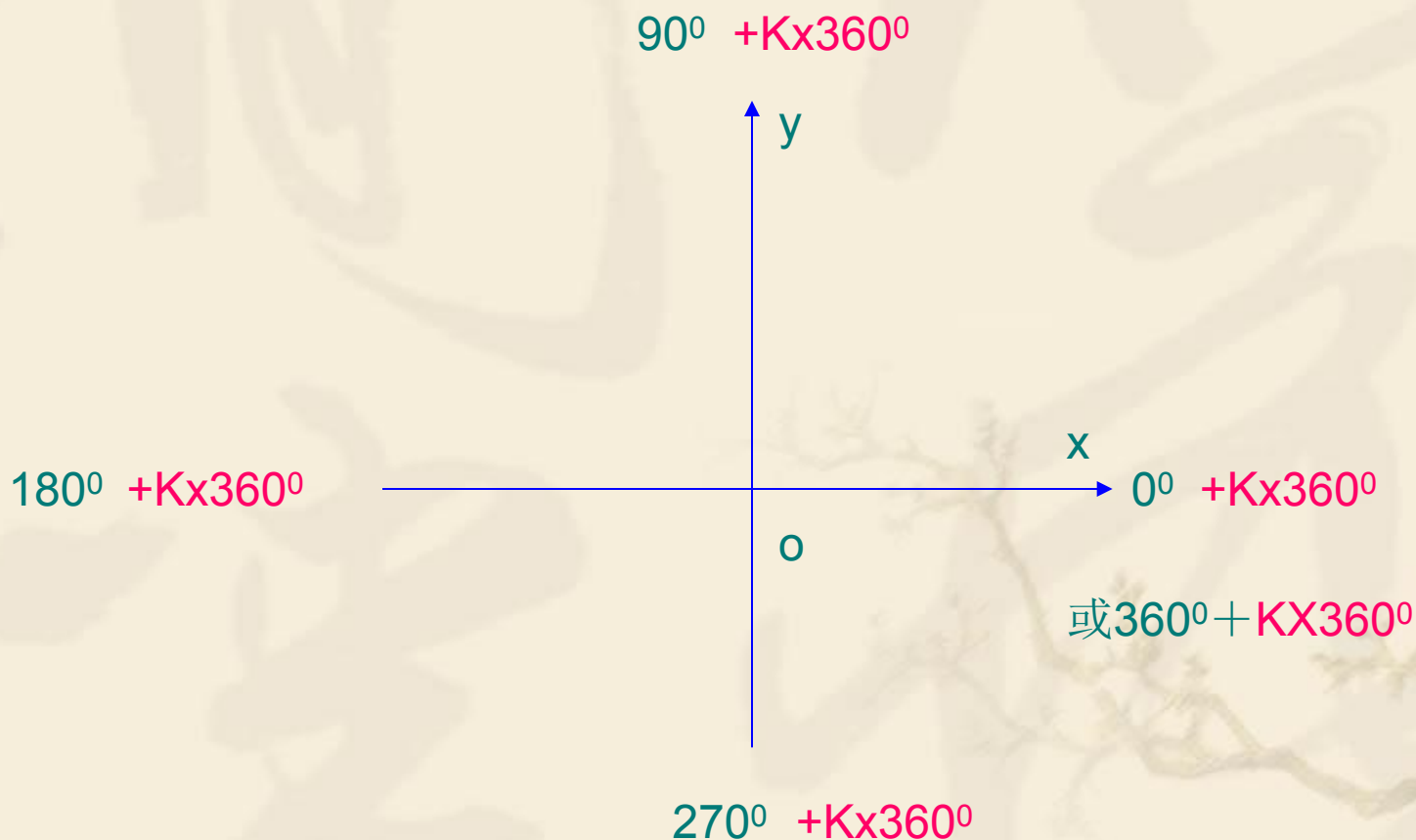
与 a 终边相同的角的一般形式为

$$a + K \times 360^{\circ}, \quad K \in \mathbb{Z}$$

$$S = \{ \beta \mid \beta = a + k \times 360^{\circ}, K \in \mathbb{Z} \}$$

例1、终边落在坐标轴上的角

❖ 终边落在坐标轴上的情形



例2 写出终边落在y轴上的角的集合。

❖ 解：终边落在 y 轴正半轴上的角的集合为

$$S_1 = \{\beta \mid \beta = 90^\circ + K \cdot 360^\circ, K \in \mathbb{Z}\}$$

$$= \{\beta \mid \beta = 90^\circ + 2K \cdot 180^\circ, K \in \mathbb{Z}\}$$

$$= \{\beta \mid \beta = 90^\circ + 180^\circ \text{ 的偶数倍}\}$$

终边落在 y 轴负半轴上的角的集合为

$$S_2 = \{\beta \mid \beta = 270^\circ + K \cdot 360^\circ, K \in \mathbb{Z}\}$$

$$= \{\beta \mid \beta = 90^\circ + 180^\circ + 2K \cdot 180^\circ, K \in \mathbb{Z}\}$$

$$= \{\beta \mid \beta = 90^\circ + (2K+1) \cdot 180^\circ, K \in \mathbb{Z}\}$$

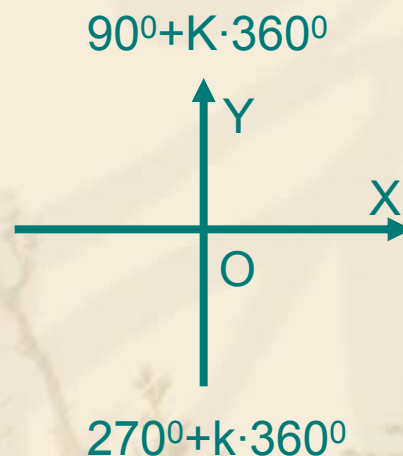
$$= \{\beta \mid \beta = 90^\circ + 180^\circ \text{ 的奇数倍}\}$$

所以 终边落在 y 轴上的角的集合为

$$S = S_1 \cup S_2 = \{\beta \mid \beta = 90^\circ + 180^\circ \text{ 的偶数倍}\} \cup \{\beta \mid \beta = 90^\circ + 180^\circ \text{ 的奇数倍}\}$$

$$= \{\beta \mid \beta = 90^\circ + 180^\circ \text{ 的整数倍}\} = \{\beta \mid \beta = 90^\circ + K \cdot 180^\circ, K \in \mathbb{Z}\}$$

$$\begin{aligned} & \{\text{偶数}\} \cup \{\text{奇数}\} \\ &= \{\text{整数}\} \end{aligned}$$



例2 写出终边落在 x 轴上的角的集合。

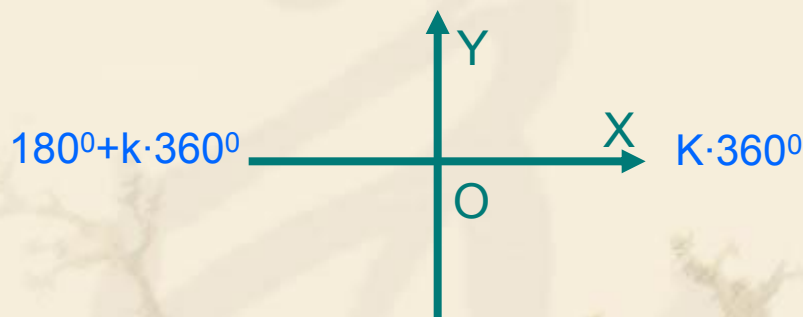
❖ 解：终边落在 x 轴正半轴上的角的集合为

$$\begin{aligned} S_1 &= \{\beta \mid \beta = 90^\circ \cdot 2K, K \in \mathbb{Z}\} \\ &= \{\beta \mid \beta = 90^\circ \cdot 2K \cdot 180^\circ, K \in \mathbb{Z}\} \\ &= \{\beta \mid \beta = 90^\circ \cdot 180^\circ \text{ 的偶数倍}\} \end{aligned}$$

$$\begin{aligned} &\{\text{偶数}\} \cup \{\text{奇数}\} \\ &= \{\text{整数}\} \end{aligned}$$

终边落在 x 轴负半轴上的角的集合为

$$\begin{aligned} S_2 &= \{\beta \mid \beta = 180^\circ \cdot 2K, K \in \mathbb{Z}\} \\ &= \{\beta \mid \beta = 180^\circ \cdot 2K \cdot 180^\circ, K \in \mathbb{Z}\} \\ &= \{\beta \mid \beta = 90^\circ + (2K+1) \cdot 180^\circ, K \in \mathbb{Z}\} \\ &= \{\beta \mid \beta = 90^\circ \cdot 180^\circ \text{ 的奇数倍}\} \end{aligned}$$



所以 终边落在 x 轴上的角的集合为

$$\begin{aligned} S &= S_1 \cup S_2 = \{\beta \mid \beta = 180^\circ \text{ 的偶数倍}\} \cup \{\beta \mid \beta = 180^\circ \text{ 的奇数倍}\} \\ &= \{\beta \mid \beta = 180^\circ \text{ 的整数倍}\} = \{\beta \mid \beta = K \cdot 180^\circ, K \in \mathbb{Z}\} \end{aligned}$$



❖ 小结:

1.任意角的概念

正角：射线按**逆时针**方向旋转形成的角
负角：射线按**顺时针**方向旋转形成的角
零角：射线**不作**旋转形成的角

2.象限角

- 1)置角的顶点于**原点**
- 2)始边重合于**X轴**的**非负**半轴
- 3)**终边**落在**第几象限**就是第几象限角

3.终边与角 α 相同的角

$$\alpha + K \cdot 360^\circ, \quad K \in \mathbb{Z}$$